author to have read the very latest things published on a subject while an acquaintance with the views of the classical old authorities is considered unnecessary. It appears that Dr. Wise did not intend to present his views or criticisms of and his answers to the latest biblical investigations. It may even be that he is not familiar with many of them. Granting this to be a fault of his book it is, nevertheless, refreshing to us to find an author who has actually read and is excellently familiar with all the old sources of the subject he is writing upon. κρς.

THE FOUNDATIONS OF GEOMETRY. By *Edward T. Dixon.* Cambridge (Eng.): Deighton, Bell & Co. 1891.

This work is divided into three parts, the first containing such doctrines of psychology and logic as the author deems sound and useful for his purposes, the second exhibiting the author's "subjective theory of geometry deduced from the two fundamental concepts *position* and *direction*," and the third "on the applicability of the foregoing subjective geometry to the geometry of material space."

In his preface the author expresses his desire that those who criticise his work shall "consider categorically" certain questions relating to his theory of definition, to the definitions and axioms prescribed by him, to his proofs of propositions and to the "objective applications" of his three axioms.

Geometry may be studied for two distinct purposes, neither of which necessarily involves the other. Unless the aim is mainly the discipline of the logical faculty, it is plainly a poor method of study to pore over the definitions, axioms, postulates, theorems, problems, and demonstrations of Euclid or any similar textbook. Practical resources and geometrical information can be acquired much better and more rapidly by a course of mechanical drawing with here and there a more or less loose explanation of the grounds and reasons that warrant the geometrical doctrines, than by means of the Euclidian course. Under such a method of instruction the student would rarely feel any real doubt as to the truth of his geometrical knowledge.

But where the paramount aim is the training of the reason the Euclidian rigor is all important. Hence the perfection of that method by the discovery and certification of the ultimate grounds on which, and the principles by which, it may be unfolded systematically and in necessary and sufficient sequence without presumption or fallacy, is an object of the most momentous concern to science, to philosophy, and to culture in general. For it is well known that however good an account elementary geometry may give of its superstructure the reports given of its foundations are all very far from satisfactory.

Repeated and strenuous efforts have been made, and by the most competent of our race, to discover and certify the true state of the case in respect to the geometrical foundations, in order that the whole edifice of that science shall display throughout the same thoroughgoing necessity and sufficiency that distinguishes it in general.

The author of the work under review is persuaded that he is now able to perform this so desirable service. He avers his belief that the system of geometry he "has set forth in this book is *logically sound* and that consequently the more it is "discussed and criticised, the more firmly will it become established." He takes his stand upon two fundamental concepts, *position* and *direction*, which he defines not explicitly but "implicitly." This leads us to consider his first question and his theory of definition.

The embarrassments that involve the foundations of elementary geometry are mainly, if not wholly, those which involve the general problems of definition. Now a definition is the certification of the purport of a name by means of a statement or a conspiracy of statements necessary and sufficient to that end. But names are constituents absolutely necessary for the formation of any statement, so that the above definition of a definition may be restated thus: A definition is the certification of the purport of one name by means of other names, necessary and sufficient to certify the purport of the one defined. Evidently then, definition can only lead us from name to name in unending process, or to some undefinable name, or to some name that we choose to leave undefined; and the question arises, on what sort of names shall we take our stand as ultimate grounds?

Our author answers this question as follows: "The propounder of a scientific "theory is not of course expected to teach his readers to speak, it is only necessary "for him to define the terms peculiar to his science, or those to which he wishes to "attach peculiar meanings. He may therefore assume that the meanings of all "other words are known to his readers."

He then propounds that "all that is logically required for a definition is one "or more assertions with regard to the word to be defined or its attributes," provided "they are not demonstrably incompatible with each other."

Although our author conceives that logical competence requires no more than this for a good definition, he yet goes on to remark, that "if the definition is to "form the basis of a deductive science it is further advisable that the assertions "should be independent," and that "where it is required to define a term whose "denotation is already known, it is further necessary not only that the assertions "should be commonly accepted as true with respect to it, but that they should restrict the meaning of the term exactly to its accepted denotation, neither more "nor less, and should do so in the simplest manner that can be devised."

It is upon this theory of definition that our author requests of his critics a "categorical" answer to his first question, "Do you accept the requirements I have "laid down for a logical definition? (If not please state which of them you object "to, why you object to it, and what you would propose to substitute for it.)"

Since it is a "categorical" answer that is requested and since also it is the matter of definition that is put in issue, we wish that our author had been more definite and had made his propositions better issuable, for we must protest that we

regard ourselves obliged to answer to what we can best conceive to be the author's true meanings rather than to what he has explicitly said.

We do not conceive that he regards it as *necessary* to a definition that the defining assertions should be expressed "in the simplest manner that can be devised." We have also to take his use of the word "restrict" as importing completion as well as limitation, and his use of the word "requirements" as intending conditions that together are sufficient as well as necessary.

If we are right in our understanding of the meanings of our author he contemplates four cases, first, the definition of a name that has no denotation already known and that is not to form the basis of a deductive science, second, the definition of a name that has no denotation already known but which is to form the basis of a deductive science, third, the definition of a name that has a denotation already known but which is not to form the basis of a deductive science, and fourth the definition of a name that has a denotation already known and is to form the basis of a deductive science.

In this fourth case our author deems it requisite for a logical definition that there shall be made one or more assertions about the subject of definition that are not demonstrably incompatible with one another, that are independent of one another, that are commonly accepted as true in respect to the subject defined and that "restrict" the meaning of the name under definition exactly to its accepted denotation.

It seems to us that this last requirement dispenses with the necessity of all the rest. If we have provided an assertion or a set of assertions that do in fact complete and limit the meaning of the subject of definition exactly to its proper denotation that is a definition in full. It implies that the defining assertions are all consistent with one another, and in case any assertion is dependent upon one or more of the rest that is a circumstance wholly immaterial. *Utile per inutile non nocetur.*

Again, what is it to be commonly accepted as true? Does logical competence depend on the altering states of our knowledge or on the fluctuations of opinion? Was a whale logically defined as a fish before we learned that it was a mammal?

The third case allows of the application of the same comment as that made upon the fourth. But in the first and second cases the doctrines of the author as well as his suppositions are very notable. He supposes the anomaly of names without any known denotation, by which he may mean those which have no application whatever. In respect to such he propounds that they may be given a logical definition by making one or various consistent assertions as applicable to them or to their attributes.

"The proof of the pudding will be found in the eating," as our author says. So let us say that a troft may be perceived whenever our attention is excited, and that trofts are of multitudinous variety. Do these assertions constitute a logical definition? It is a prime requisite for a definition that the defining assertion or assertions shall have a meaning, which is the same as to say that names must be

employed that are already significant. These significant names must be so used that the intellectual sensibility shall be excited to perceive in a determinate way that which is intended to be defined. In other words, sense and not nonsense must be produced in the mind that considers the definition. Perhaps, however, our author intends such words as electricity, or spirit, or energy.

Because of the considerations above indicated and others we cannot accept the author's requirements for a logical definition as a whole. Some of them are in some of his cases unnecessary, while taken together they supply no new means whereby to solve the several problems of definition.

The author's subjective theory of geometry is plainly the outgrowth of his confidence in the solvent power of the concept of direction as a prime datum of geometry.

Everything of consequence in his essay depends upon the worth of this concept as a geometrical foundation. Considering the disparagement that has been visited upon that concept by numerous writers of good geometrical rank we naturally look for considerations tending to remove the discredit that has befallen that notion. Instead however of this we find the most palpable set of circular definitions. Direction is defined by direction in the most distracting way, thus:

"(*1*) A direction may be conceived to be indicated by naming two points as the "direction from one to the other."

The inaptitude of the term direction for use in geometry is rooted in its ambiguous purport. As commonly used it means at least three distinct but closely associated notions which become confused in thought and expression unless the most solicitous care is taken to distinguish them. When we speak of the direction of one point from another or of the direction from one point to another we mean the straight off-ness or from-ness or to-ness which one bears to the other ; in other words a relation of separation and straight mediation. When again we speak of the direction of a motion we intend the indefinite straight sense of its procession, which is not a relation but an attribute of the motion. When still again we speak of the direction of a line we mean its straight *lay* as compared or as comparable with other actual or possible correllates which is again a relation but not necessarily the same relation as that that obtains between two points.

In all these meanings the notion of straightness is involved, and could we say in lieu of straightness first directness and then direction and holding fast in thought this sense of the word, make a noun of it, so that a direction would intend the same as a straightness and no more, it might obtain a useful geometric term and notion.

To define it we might first define a line thus : A line is a space boundary that is indefinitely long but not otherwise of any extent. Then, a direction is a line such that between the points that bound any assigned parcel of it no copy of said parcel is possible.

But direction purports to our author the second of the meanings above set forth, namely, the indefinite straight sense of the procession of a motion. Definite parcels of a direction thus understood are identical with vectors.

Now the notion of straightness is after the notions of point and line the most fundamental one of geometry and the one which is altogether the most prominent and useful. It is the necessary means for any definition of a vector or of the notion which our author deems so important. As straightness is attributable only to lines and long things which a line may represent it makes no difference whether we define straightness or a straight line, but a masterful performance of this definition is absolutely necessary before the foundations of geometry can be abidingly certified.

Our author defines a straight line thus: "A straight line is a continuous series "of points extending from each of them in the same two directions." What kind of a thing a continuous series of points may be we are not told but as a point is defined to be "a portion of matter so small that for the purpose in hand variations "of positions within it may be neglected" we take it that a straight line is a continuous series of particles of matter. The "purpose in hand" in this case must of course be the purpose of geometry.

In defining an angle our author first lays down that "The difference between "two directions is called their *inclination* to one another" and then "The measure "of an inclination is called an *angle*."

Considering that it is the doctrine of the author that every straight line has two contrary directions the measure of whose inclination is an angle of one hundred and eighty degrees, we imagine a northeast southwest line cutting an east west line and wonder if the right hand upper angle is really two angles according to whether or not the directions both pass to the left or both pass to the right or pass one to the left and the other to the right.

Were this an ordinary work we might regard it as due to the author to notice the many excellencies which characterise it, in spite of the defects which we notice. But as our author evidently realises, the eminent dignity of the topic challenges and its singular importance demands unsparing criticism. He who offers to instruct the world on the foundations of geometry draws his sword and throws away the scabbard, and like a doughty champion he will scorn to accept any favor, prizing only such success as he shall take at the point of an efficacy of treatment that conquers all competent and candid criticism.

Stringent as are such terms of contest an author who is a worthy competitor in the field of geometric research can be well content with them in the perception that the very same conditions apply in full force to the comments of his critics.

The author is undoubtedly an able man and a close thinker. He has concentrated his mind upon a work that is worth the energy of a lifetime. But we must confess our judgment to be that in spite of his capacity and evident devotion he has come short of the high result to which he has aspired. ρσλ.